

Indian Statistical Institute, Bangalore Centre

B.Math III Year, First Semester

Mid-Sem Examination

Differential Equations

Time: 3 Hours

September 14, 2012

Instructor: C.R.E. Raja

Total Marks : 50

Answer any five, each question carries 10 marks, total marks: 50

1. (a) Prove that the solution of $y' + P(x)y = Q(x)$ is $y = e^{-\int P dx} (\int Q e^{\int P dx} dx + c)$ where c is a constant.
(b) Solve the equation $xy' = y + 2xe^{-y/x}$.
2. (a) Let $Mdx + Ndy = 0$ be the given ODE. If $g = \frac{M_y - N_x}{2xyN - x^2M}$ is a function of $z = x^2y$, does the equation has an integrating factor? Justify your answer.
(b) Solve $x^2y'' = 2xy' + (y')^2$.
3. (a) If y_1, y_2 are solutions of $y'' + P(x)y' + Q(x)y = R(x)$ on \mathbb{R} where P, Q, R are continuous function on \mathbb{R} . Prove that $\{x \in \mathbb{R} \mid y_1(x) = y_2(x)\}$ is countable.
(b) Is there a ode $y'' + P(x)y' + Q(x)y = 0$ that has $f(x) = x^3$ and $g(x) = x^2|x|$ as solutions on $[-1, 1]$ where P, Q are continuous functions on $[-1, 1]$. Justify.
4. Let y_1, y_2 be two solutions of $y'' + P(x)y' + Q(x)y = 0$ and W be the Wronskian of y_1 and y_2 .
(a) Prove that W is always zero or never zero.
(b) Prove that W has a zero if and only if y_1 and y_2 are linearly dependent solutions.
5. Let y be a solution of the ode $y'' + py' + qy = 0$ where p, q are constants.
(a) Prove that y is infinitely many times differentiable.
(b) y and all its derivatives are also solutions of $y'' + py' + qy = 0$.
(c) Solve the equation $2x^2y'' + 10xy' + 8y = 0$.
6. (a) Find the general solution of the following system of ode $x'(t) = -x - y$ and $y'(t) = x - 2y$.
(b) Find the general series solution of the equation $y'' - 2xy' + 2py = 0$ where p is a constant.

7. (a) Find a solution of the ode $(1 + x^2)y'' + 2xy' - 2y = 0$ using power series method.
- (b) Show that the equation $x^2y'' + xy' + (x^2 - 1)y = 0$ has only one Frobenius series solution and find the solution.